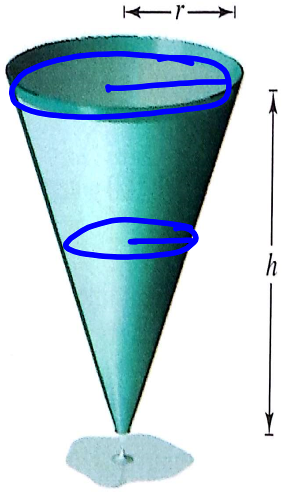
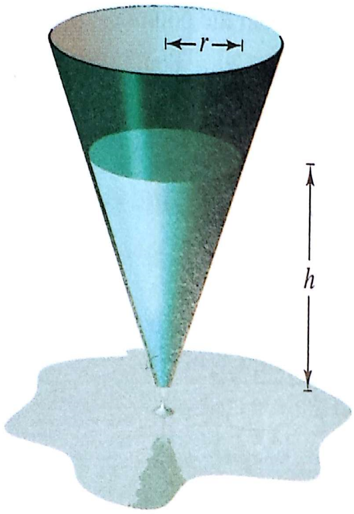
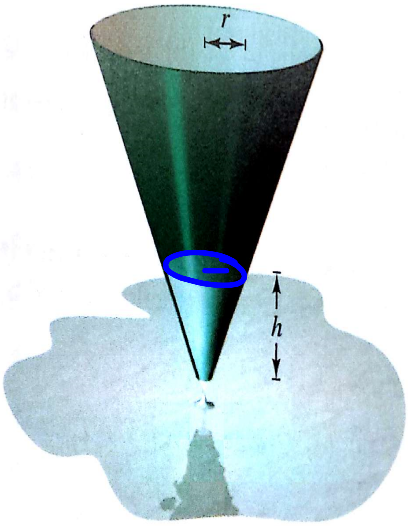
Related Rates

Find the rates of change of *two or more related variables* that are changing with respect to time.



$\frac{1}{3}\pi \underline{r}^2 \underline{h} = V$





$$\frac{d}{dt}[v] = \frac{d}{dt}\left[\frac{\pi}{3}r^2 h\right]$$

$$\frac{dv}{dt} = \frac{\pi}{3} \frac{d}{dt}\left(r^2 h\right)$$

$$\frac{dv}{dt} = \frac{\pi}{3}\left[2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right]$$

The variables *x* and *y* are both differentiable functions of *t* and are related by the equation $y = x^2 + 3$

Find dy/dt when x=1,

given that dx/dt =2 when x=1

$y = x^2 + 3$

differentiate w/ respect to t

$\frac{d}{dt}(y) = \frac{d}{dt}(x^2+3)$

$\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\frac{dy}{dt} = 2(1)(2)$

$\frac{dy}{dt} = 4$

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles.

The radius, r, of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

$A = \pi r^2$

$\frac{dr}{dt} = 1$

$\frac{d}{dt} A = \frac{d}{dt} \pi r^2$

Find $\frac{dA}{dt}$ $r = 4$

$\frac{dA}{dt} = \pi \left[2r \frac{dr}{dt} \right]$

$\frac{dA}{dt} = 2\pi (4)(1)$

ft^2/sec

$\frac{dA}{dt} = 8\pi$ sq ft/sec

- Identify given quantities
- Identify what you are finding
- Make a sketch
- Write an equation
- Use Chain Rule, implicitly differentiate both sides *with respect to time, t.*
- Substitute back into the resulting eq. Then solve for the required rate of change.

Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

Know: $\frac{dv}{dt} = 4.5 \text{ ft}^3/\text{min}$

find $\frac{dr}{dt} =$ when $r = 2$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4\pi}{\cancel{3}}\left[\cancel{3}r^2 \frac{dr}{dt}\right]$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4.5 = 4\pi(2)^2 \frac{dr}{dt}$

$\frac{4.5}{16\pi} = \frac{dr}{dt}$

$\approx .09$ ft/min